Axion Electrodynamics in Solids

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Oxford Physics

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Two Propositions on Axion Electrodynamics

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Two Applications of Axion Electrodynamics

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Whether or not axions[1] have any physical reality, their study can be a useful intellectual exercise.

Also, it is (I shall argue) not beyond the realm of possibility that fields whose properties partially mimic those of axion fields can be realized in condensed-matter systems.

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Maxwell Lagrangian with an axion θ-term: fine structure constant α = 1/137

$$\mathcal{L}_{\rm EM} = \frac{1}{8\pi}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) + \frac{\alpha}{2\pi}\frac{\theta}{2\pi}\boldsymbol{E}\cdot\boldsymbol{B}$$

Maxwell

Axion

Disclaimer: we'll be discussing "static" axions for most of the talk, and in ordinary electromagnetism

Image credit: Wikipedia, Etsy

Charges & currents from electrons and ions = sources of $\boldsymbol{E}$ and $\boldsymbol{B}$

$$\mathcal{L}_{\rm EM} = \frac{1}{8\pi}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) - \rho A_0 - \boldsymbol{j}\cdot\boldsymbol{A}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla} A_0 - \partial_t \boldsymbol{A}$$
$$\boldsymbol{B} = -\boldsymbol{\nabla}\times\boldsymbol{A}$$

Maxwell's Equations (= Euler-Lagrange equations for A_0, $\boldsymbol{A}$)

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0 \qquad \boldsymbol{\nabla}\times\boldsymbol{E} = -\partial_t\boldsymbol{B}$$

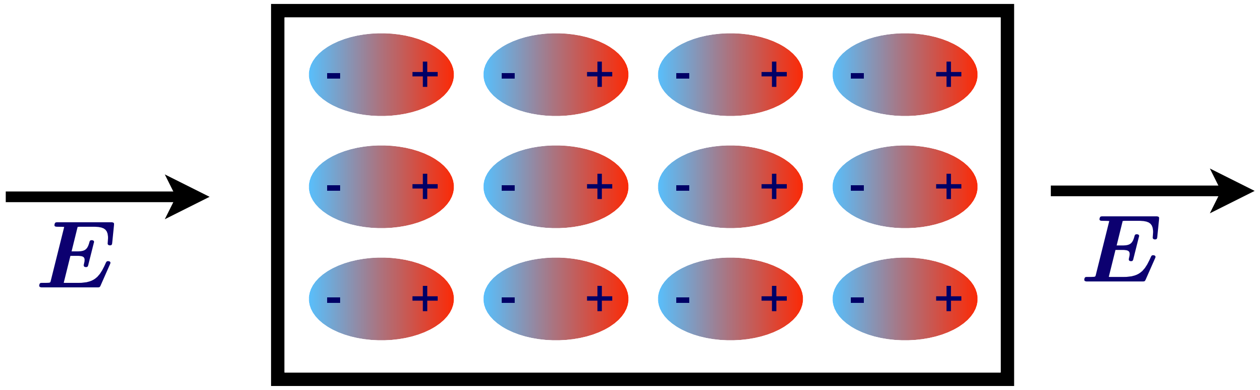
(no sources - unchanged from vacuum)

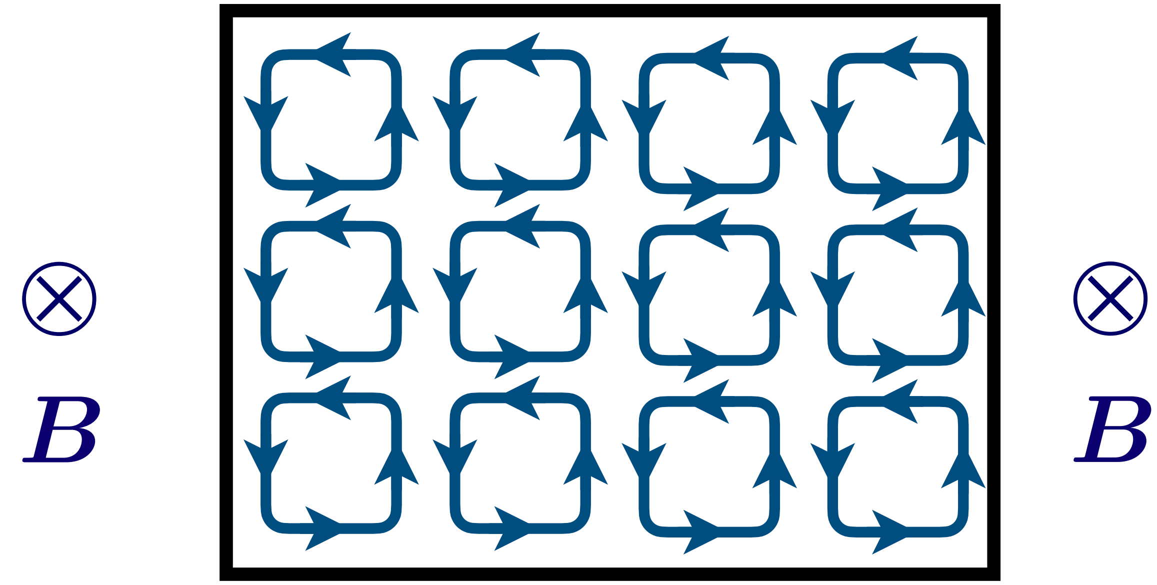
$$\boldsymbol{\nabla}\cdot E = 4\pi\rho \qquad \boldsymbol{\nabla}\times\boldsymbol{B} = 4\pi\boldsymbol{j} + \partial_t\boldsymbol{E}$$

To go further, we need a physical picture of ρ and j (e.g. metal vs. insulator)

Polarization and Magnetization

Insulators: charges/currents bound to ions ⇒ electric/magnetic dipoles fixed in space





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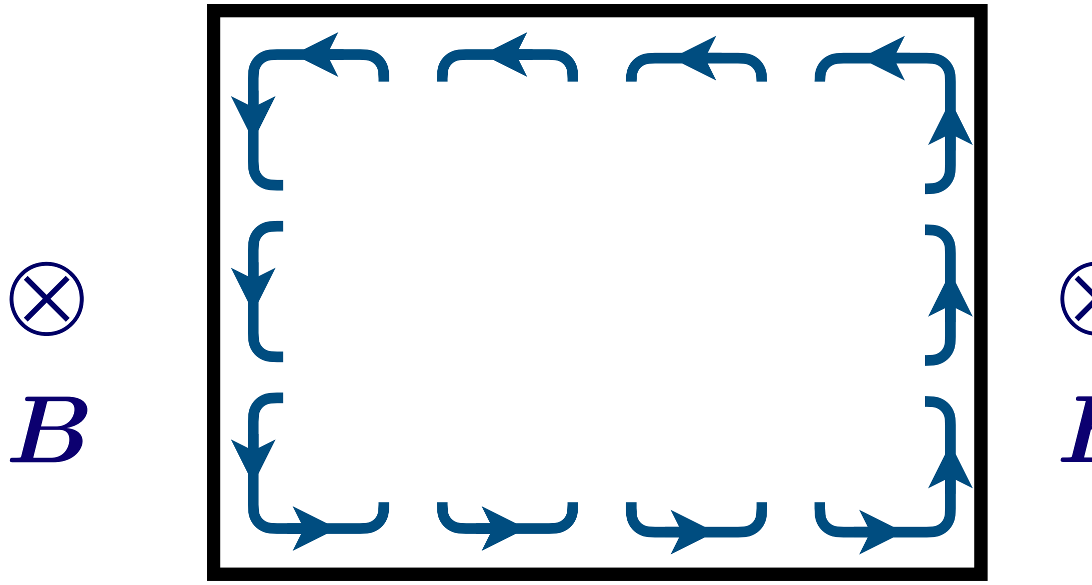

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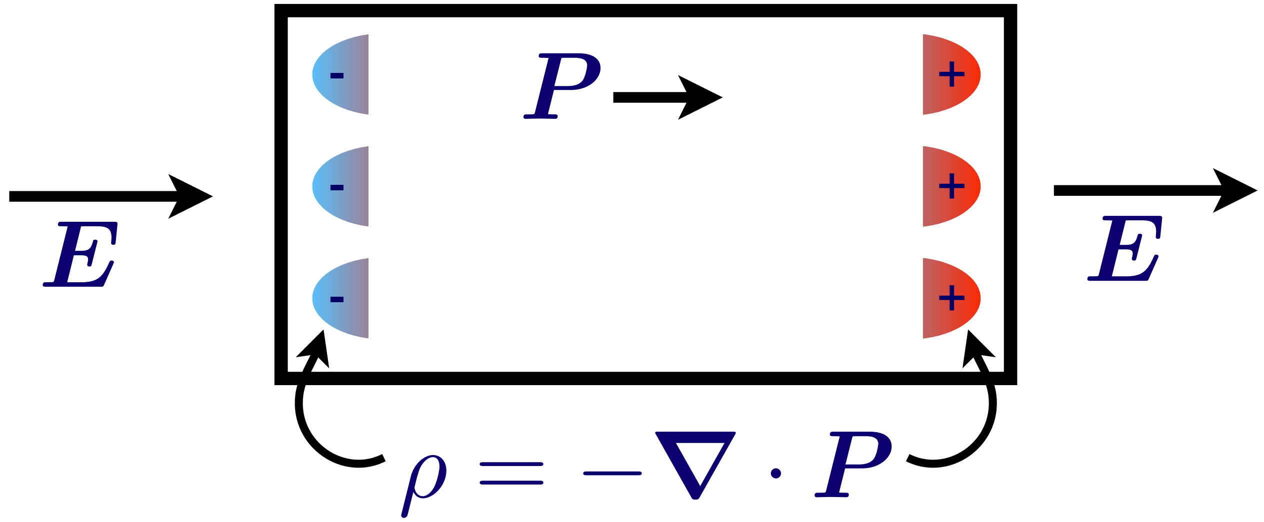

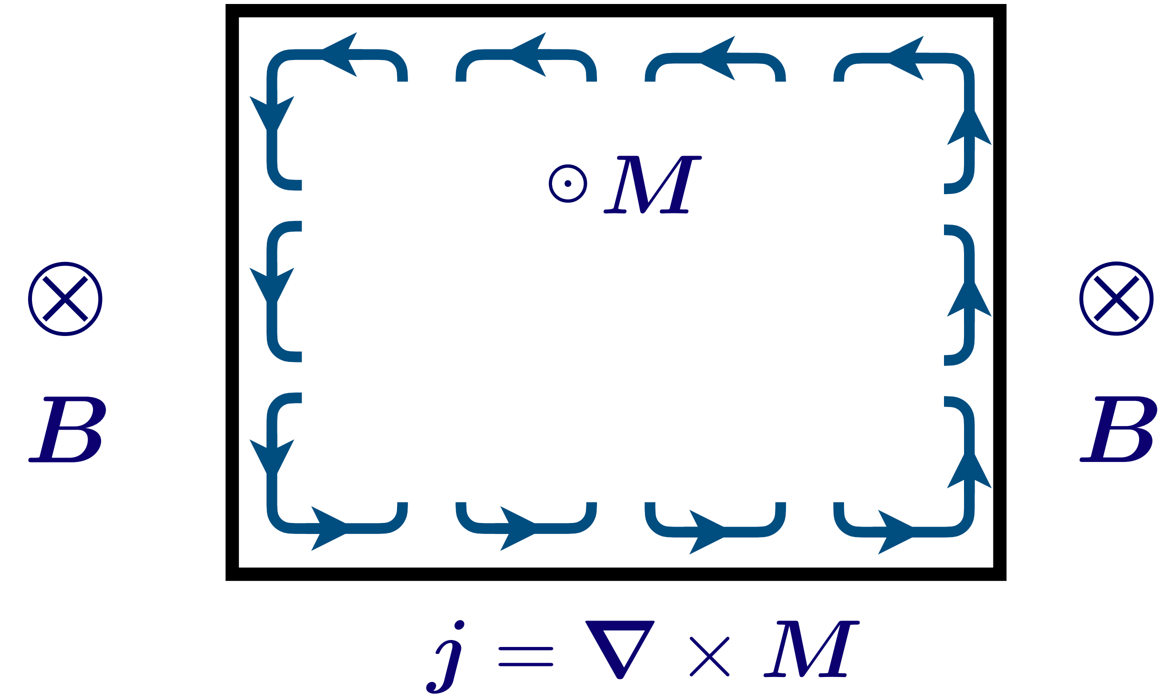

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⇒ write ρ & **j** in terms of "polarization" & "magnetization" (need $\partial_t \boldsymbol{P}$ for consistency)

$$\rho = -\boldsymbol{\nabla}\cdot\boldsymbol{P} \qquad\qquad \boldsymbol{j} = \boldsymbol{\nabla}\times\boldsymbol{M} + \partial_t\boldsymbol{P}$$

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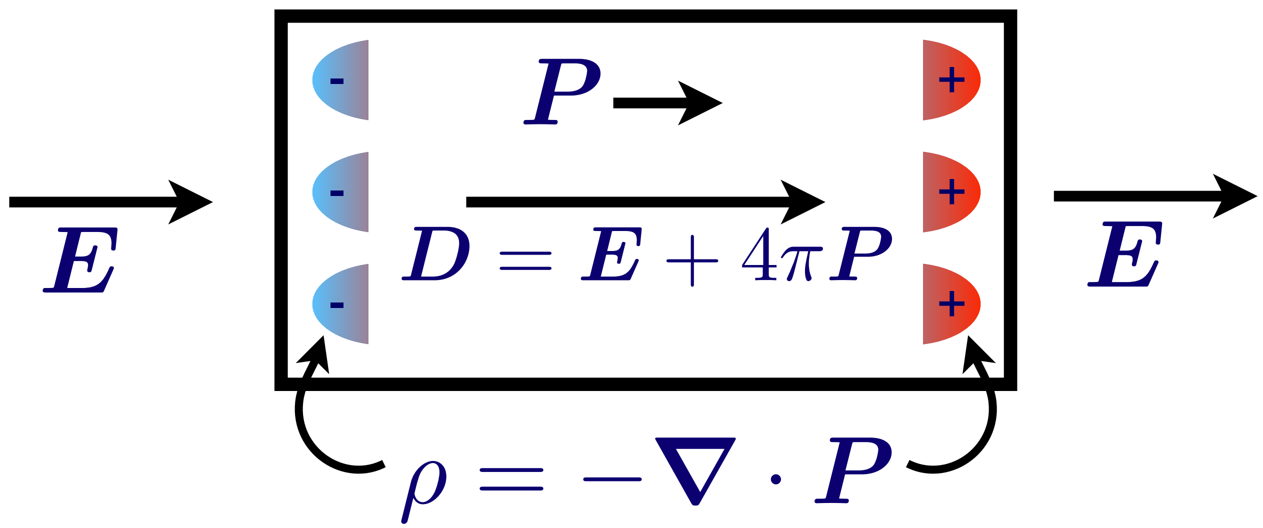

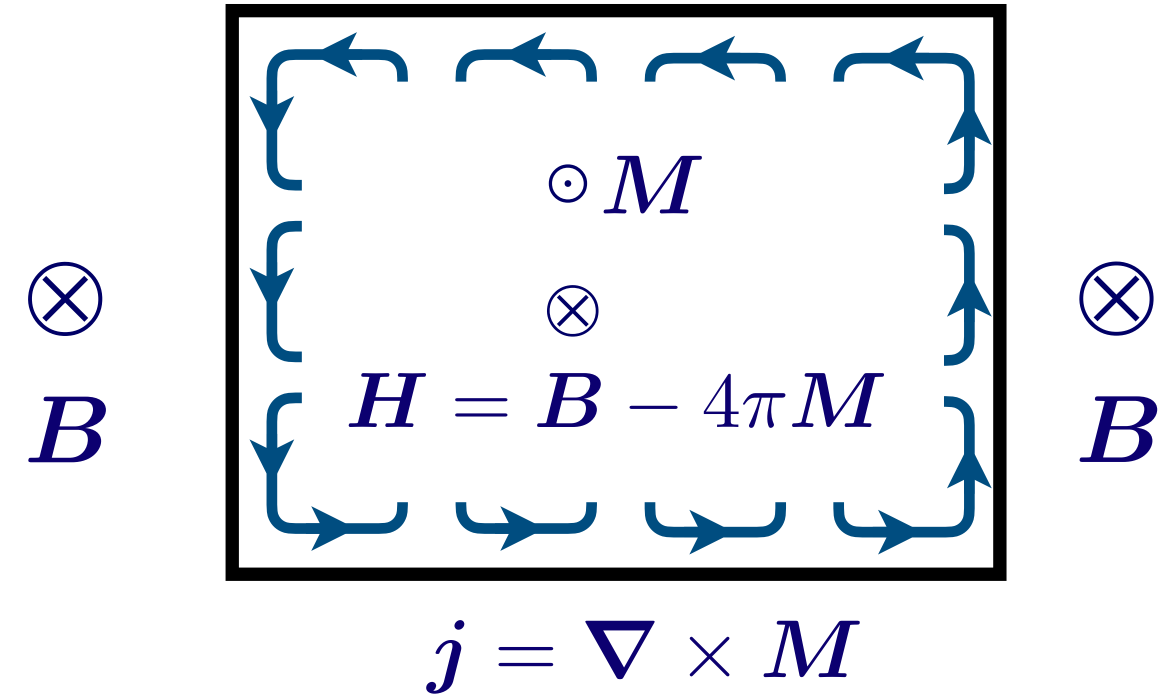

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Introduce "coarse-grained" fields averaged over atomic scales:

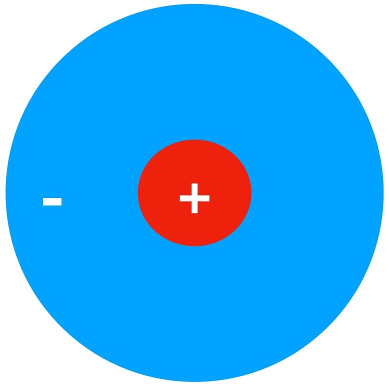
$$\boldsymbol{\nabla}\cdot(\underbrace{\boldsymbol{E}+4\pi\boldsymbol{P}}_{\boldsymbol{D}}) = 0 \qquad\qquad \boldsymbol{\nabla}\times(\underbrace{\boldsymbol{B}-4\pi\boldsymbol{M}}_{\boldsymbol{H}}) = \partial_t(\underbrace{\boldsymbol{E}+4\pi\boldsymbol{P}}_{\boldsymbol{D}})$$

"displacement field" $\boldsymbol{D}$

"magnetic field strength" $\boldsymbol{H}$

Linear Media: Insulators as New Vacua

Consider applying ***E*** to a dielectric medium (no frozen-in polarization)



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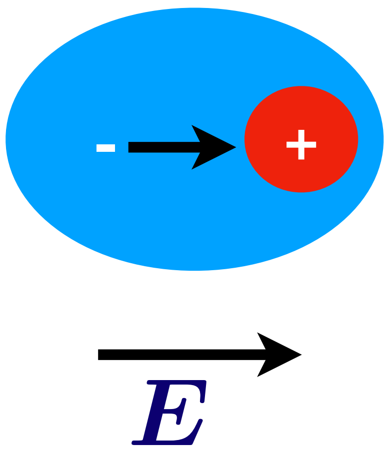

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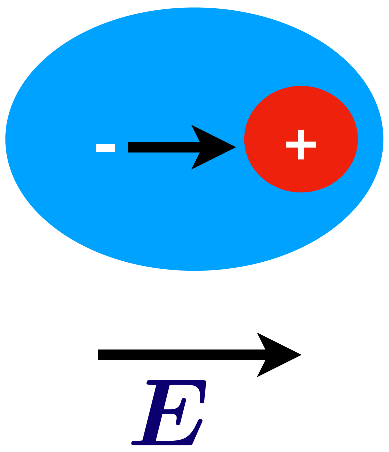

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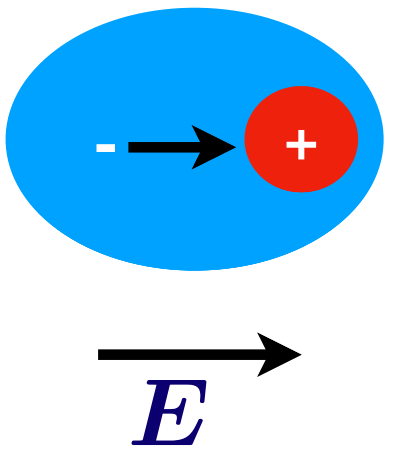

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(ϵ: dielectric constant)

$$\boldsymbol{B} = (1 + 4\pi\chi_m)\boldsymbol{H} \equiv \mu \boldsymbol{H}$$

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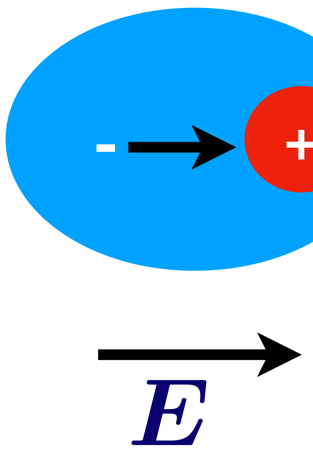

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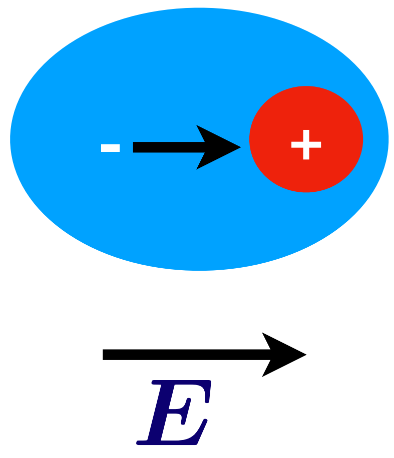

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Lesson: each insulator is effectively a new "vacuum" for electromagnetism

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The insulators we normally encounter can be recast as

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Image credit: Wikipedia, Etsy

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Axion

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$$\boldsymbol{\nabla}\cdot\left(\boldsymbol{E} + 4\pi\boldsymbol{P} - \frac{\alpha}{\pi}\theta\boldsymbol{B}\right) = 0$$

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Magnetic field induces electric polarization

Electric field induces magnetization

How could such a "magneto-electric polarizability" arise in a solid?

(need to generate a "crossed response" between ***E*** and ***B***)

Image credit: Wikipedia, Etsy

Quantum Theory of Solids

Electrons in solids: described by Schrödinger equation in periodic potential

$$\left(\frac{\mathbf{p}^2}{2M} + V(\mathbf{r})\right)\psi(r) = E\psi(\mathbf{r})$$

$$V(\mathbf{r}+\mathbf{R}) = V(\mathbf{r})$$

$\mathbf{R} \in$ lattice (assume cubic)

<u>Bloch's Theorem</u>: eigenstates = (plane wave) $\times$ (periodic function)

$$H\psi_{n\mathbf{k}}(\mathbf{r}) = E_{n\mathbf{k}}\psi_{n\mathbf{k}}(\mathbf{r}) \qquad \psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n\mathbf{k}}(\mathbf{r})$$

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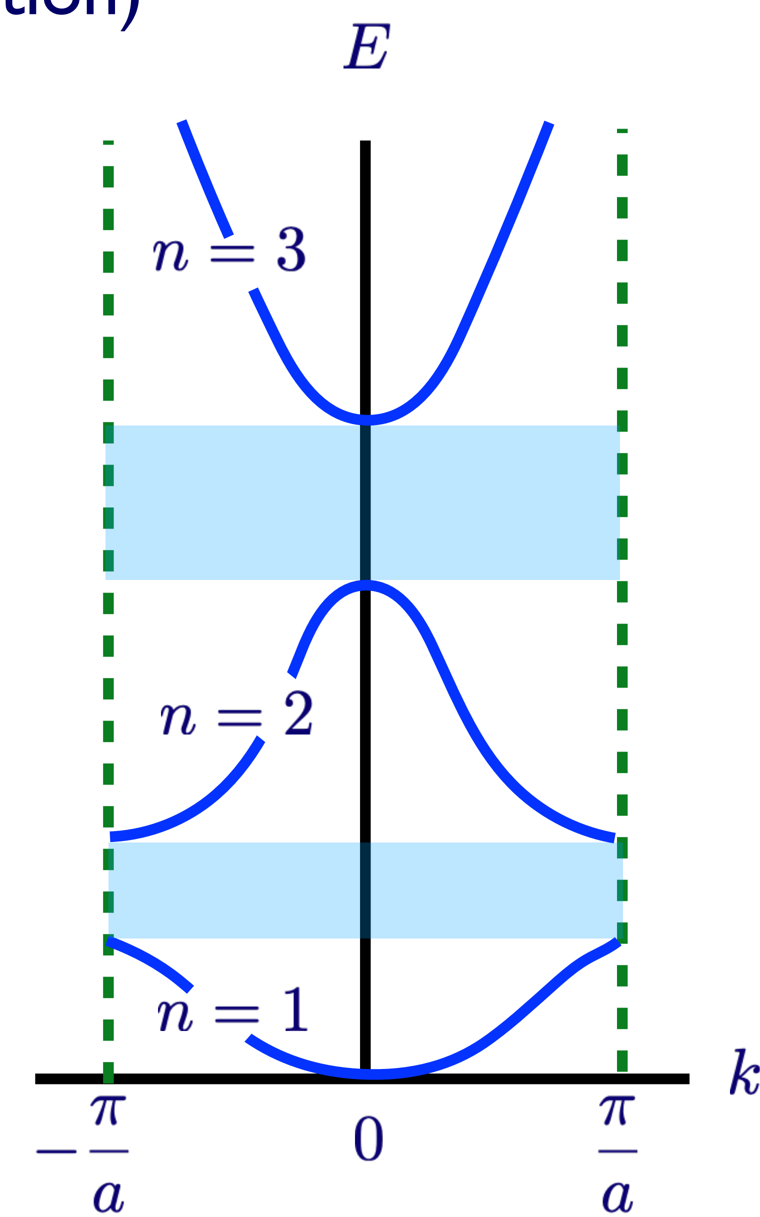

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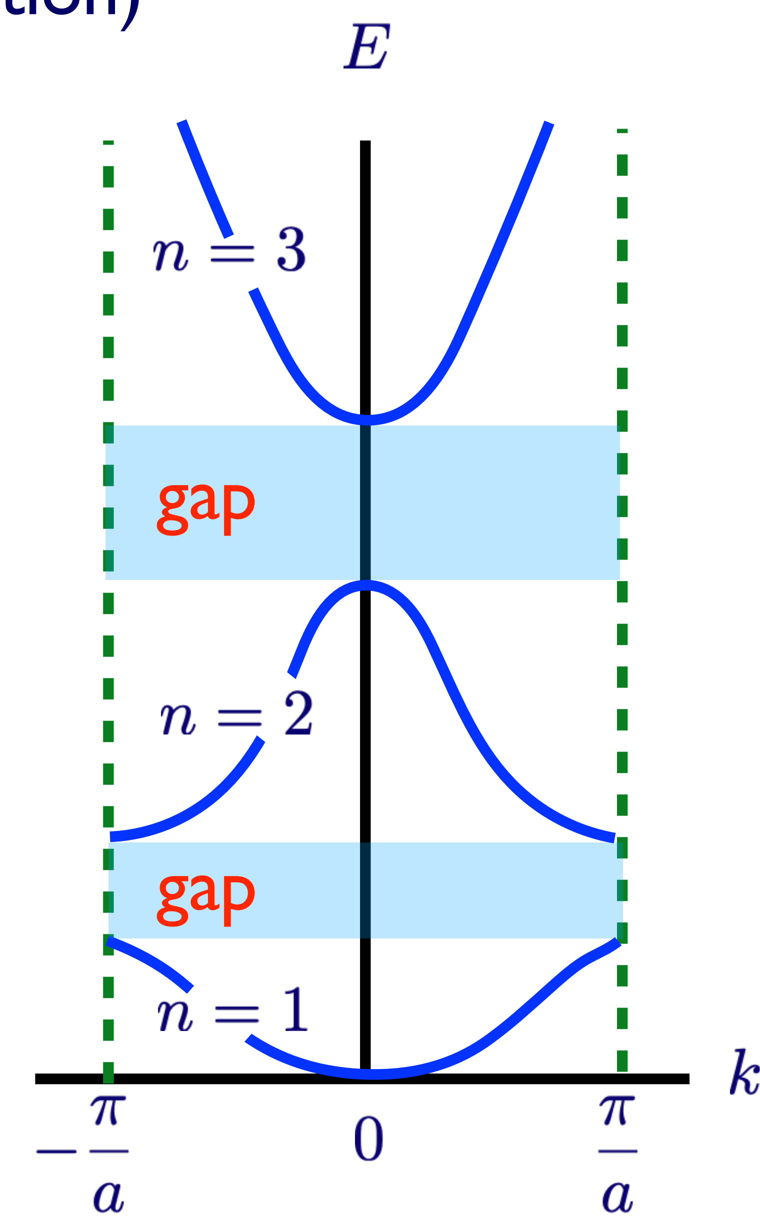

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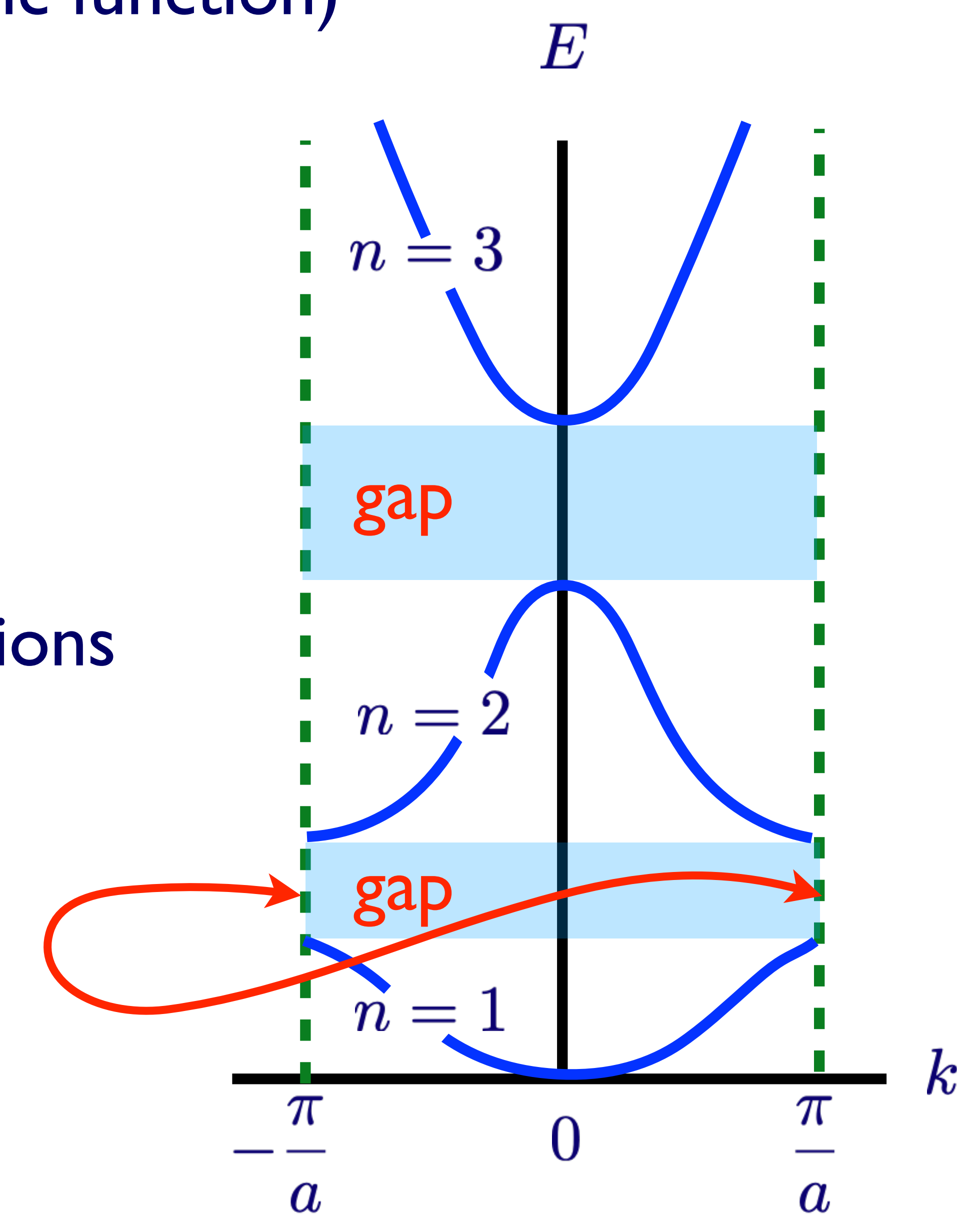

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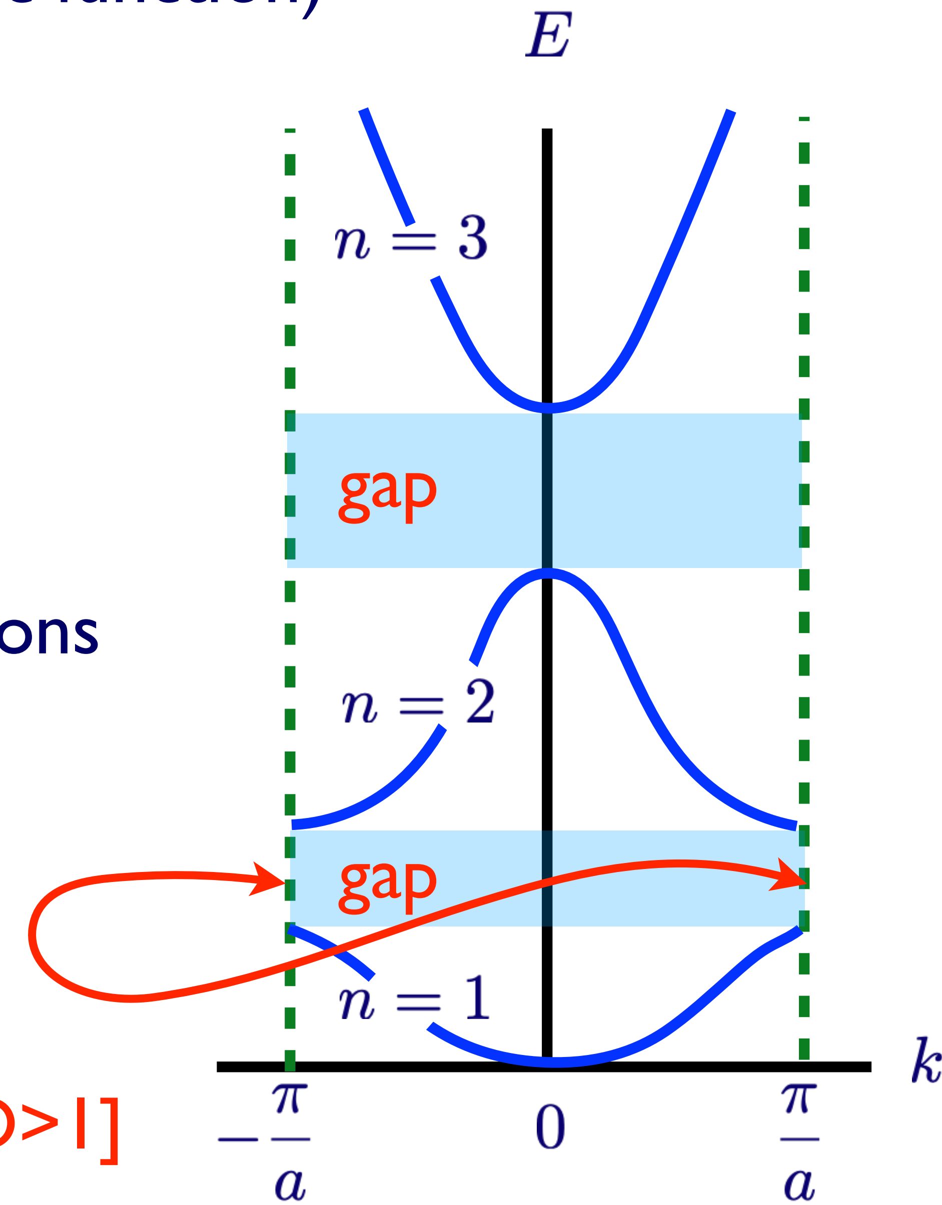

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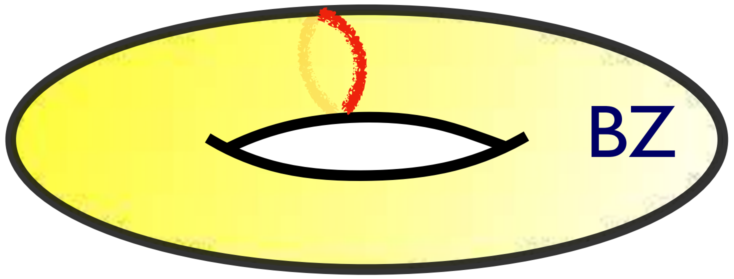
⇒ allowed *k* values = circle ["Brillouin Zone"; torus in D>1]

Topology in Solids: A Glimpse

The *phase* of the wavefunction is usually unimportant, but can play a meaningful role when it changes nontrivially over a closed loop ("Berry's phase")



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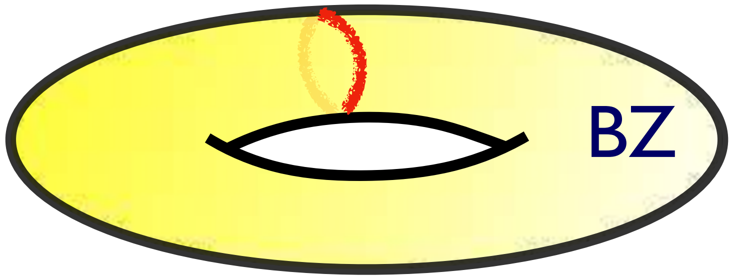

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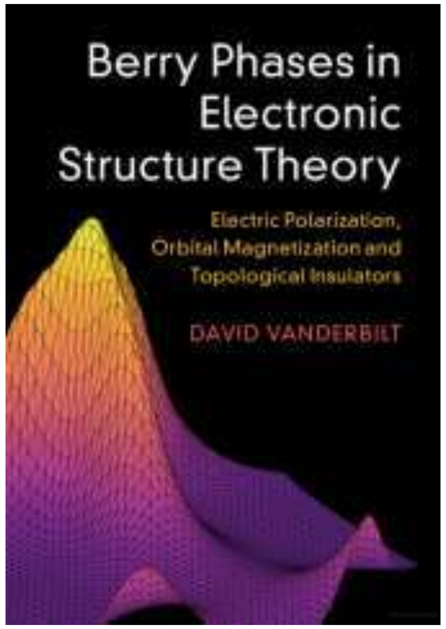
Such forces give rise to θ term [= "Chern-Simons* action"]



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$$\mathcal{A}_j^{mn} = i\langle u_{m\mathbf{k}}|\partial_j|u_{m\mathbf{k}}\rangle$$

[Qi-Hughes-Zhang '08; Essin-Moore-Vanderbilt '09]

… but *microscopic* details are gory (and take a whole textbook)

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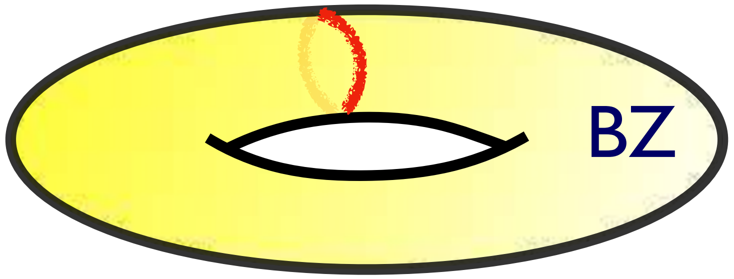

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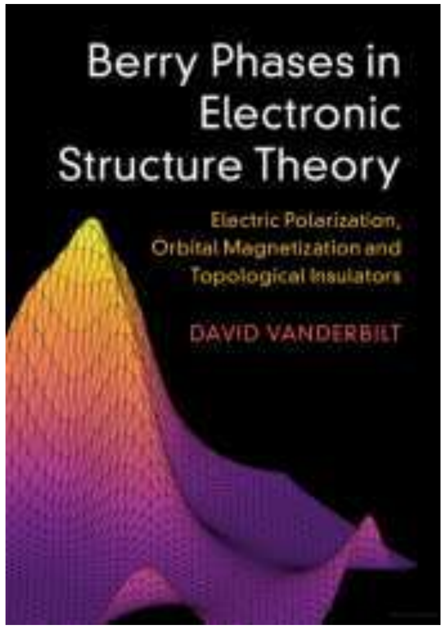

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Instead, let's "coarse-grain" and think about axion physics in solids in the spirit of "effective field theory"

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Topology, Symmetries and the θ-angle

Axion term enters quantum theory only via $e^{iS_\theta/\hbar} = e^{\frac{i}{\hbar}\frac{\theta}{2\pi}\int dt d^3x \, \frac{\alpha}{2\pi}\boldsymbol{E}\cdot\boldsymbol{B}}$

Topology of electromagnetic fields requires $e^{iS_\theta/\hbar} = e^{i\theta n}, \;\; n \in \mathbb{Z}$

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time-reversal $\quad \mathcal{T} : (\mathbf{x}, t) \to (\mathbf{x}, -t) \quad \Rightarrow \quad \begin{array}{l} \boldsymbol{E} \to \boldsymbol{E} \\ \boldsymbol{B} \to -\boldsymbol{B} \end{array}$

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Topology of electromagnetic fields requires $e^{iS_\theta/\hbar} = e^{i\theta n}, \quad n \in \mathbb{Z}$

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Only values consistent with either symmetry: $\quad \theta = 0, \pi$

Other values forbidden ⇒ θ can’t continuously vary ⇒ quantized

Symmetry can fix θ <u>independent</u> of material details!

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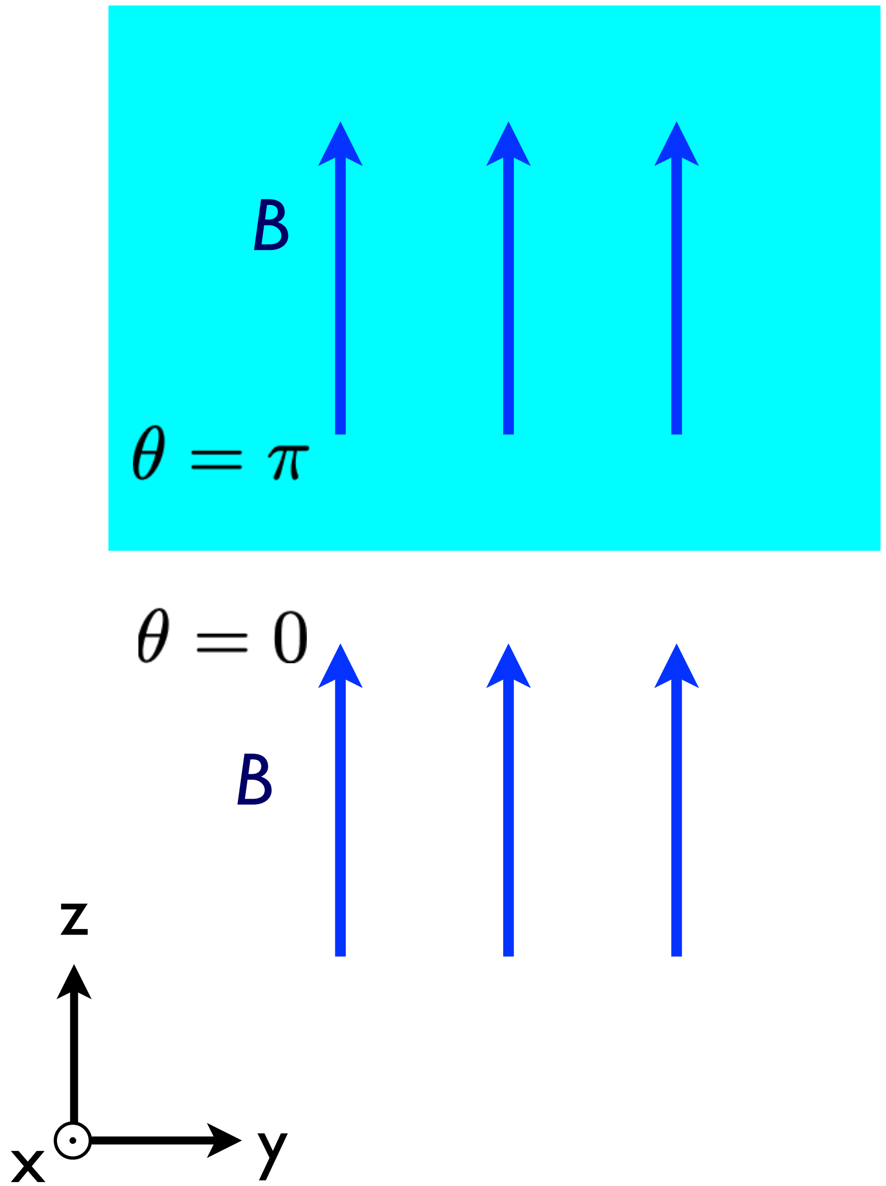
Maxwell's equations are modifed only if θ varies in space or time (e.g. at interfaces between systems with θ = 0 and θ = π)

Interfaces between θ=0 and θ=π

Consider interface between θ = 0 and θ = π and apply only *B* field as shown



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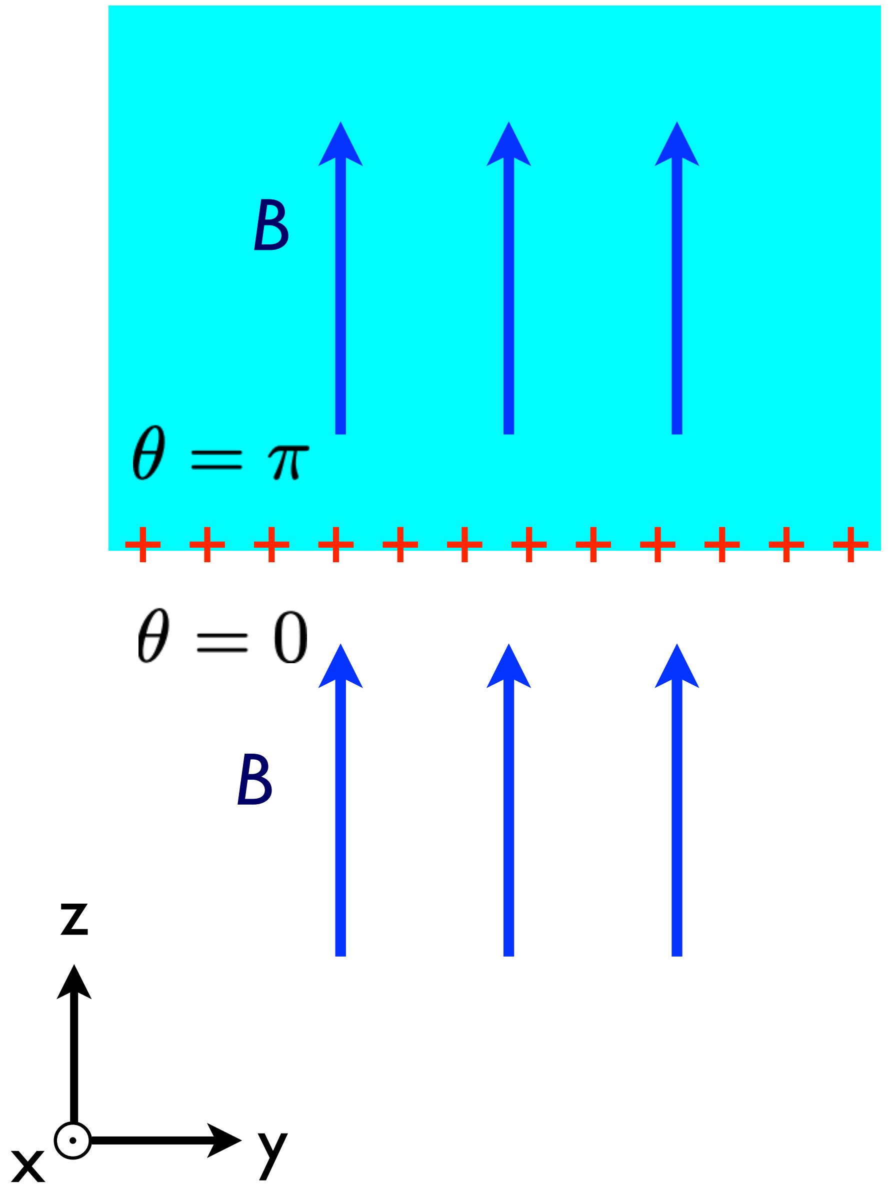

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$$\sigma = \frac{\alpha}{\pi}\boldsymbol{B}\int_{-a}^{a} dz \frac{\partial \theta}{\partial z} = \alpha \boldsymbol{B}$$

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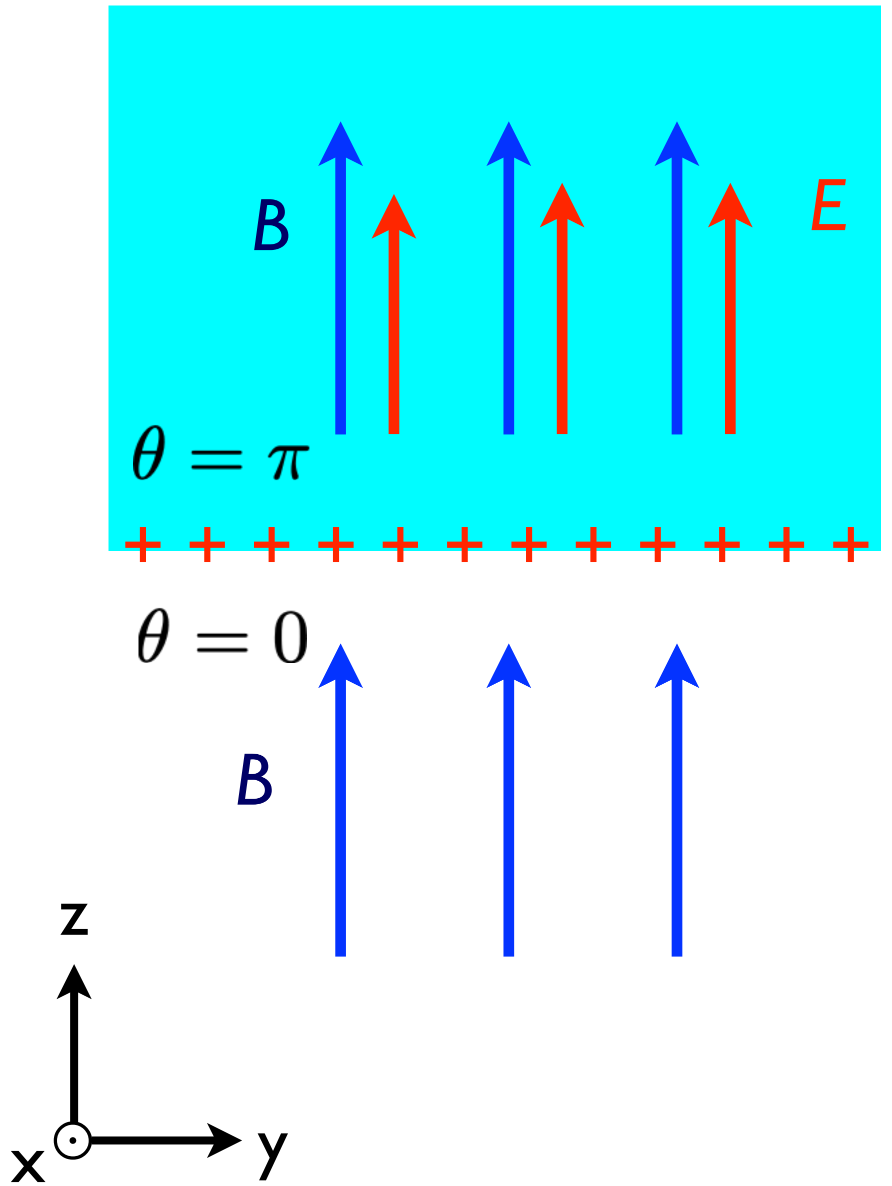

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This generates *E* parallel to *B*, *inside the material*, with strength related by fine structure constant!

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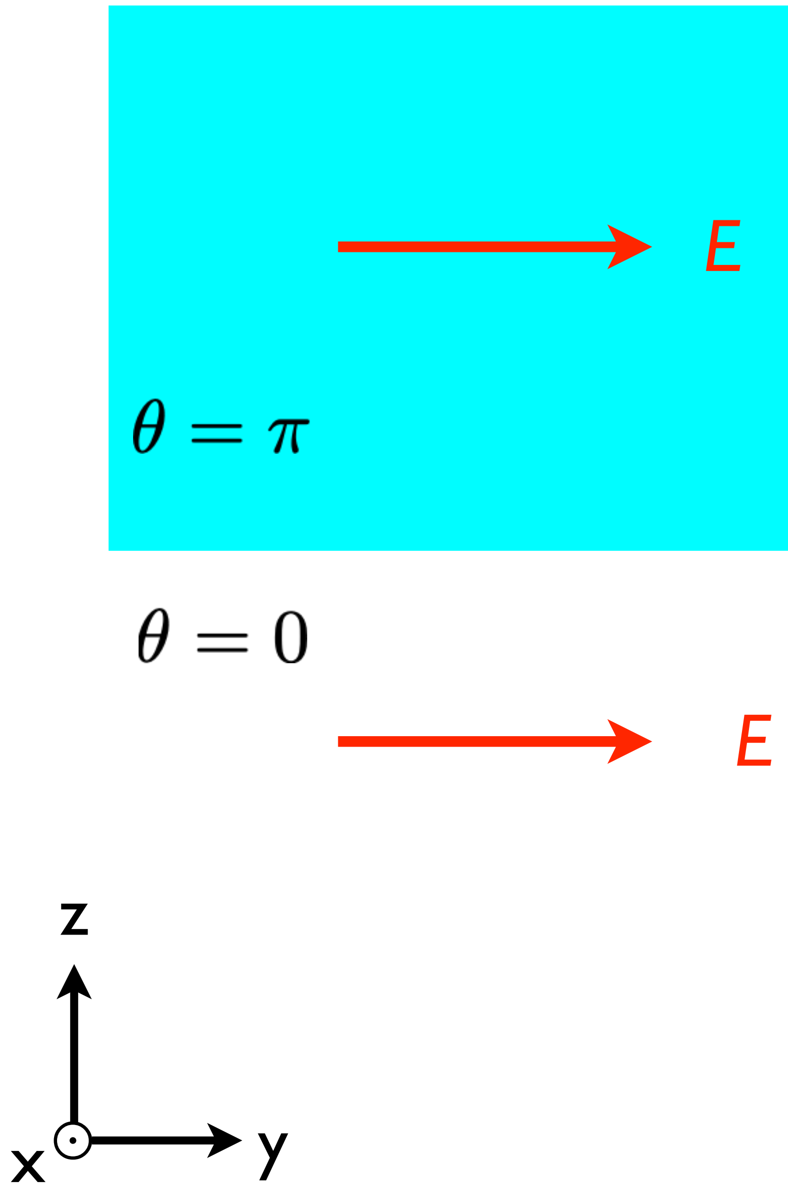

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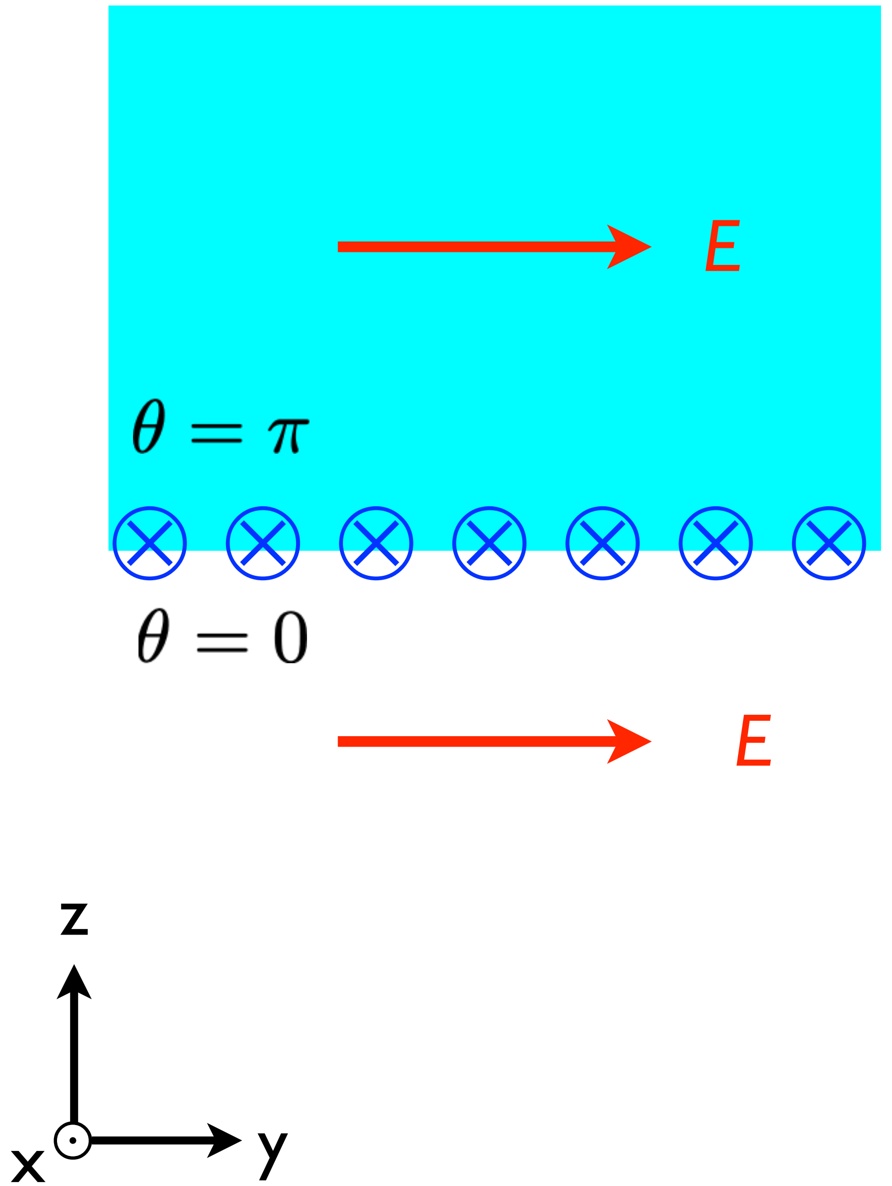

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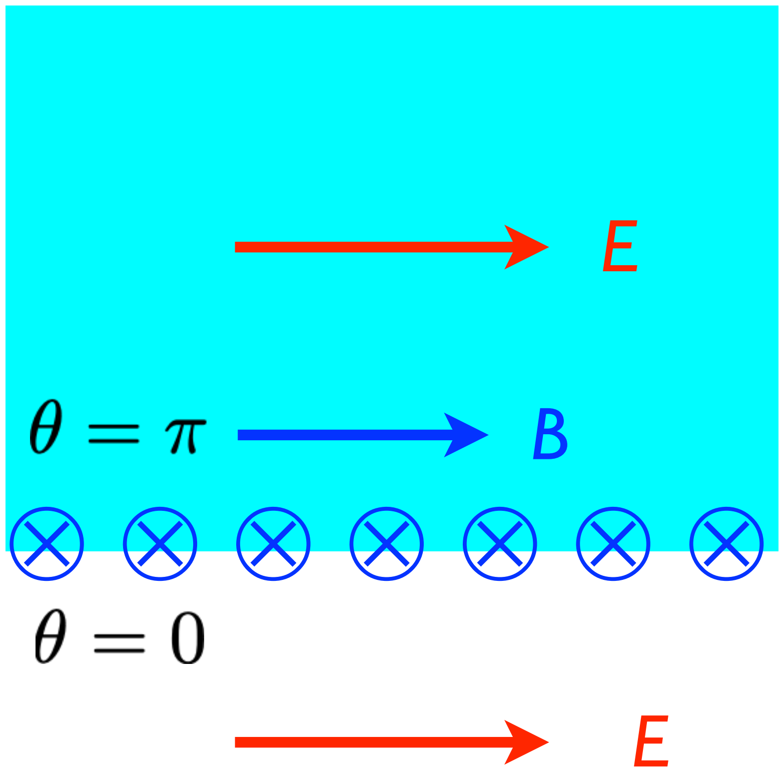

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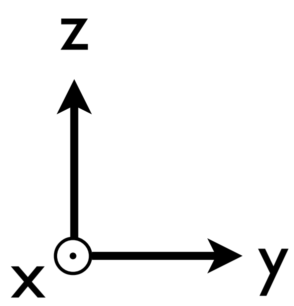

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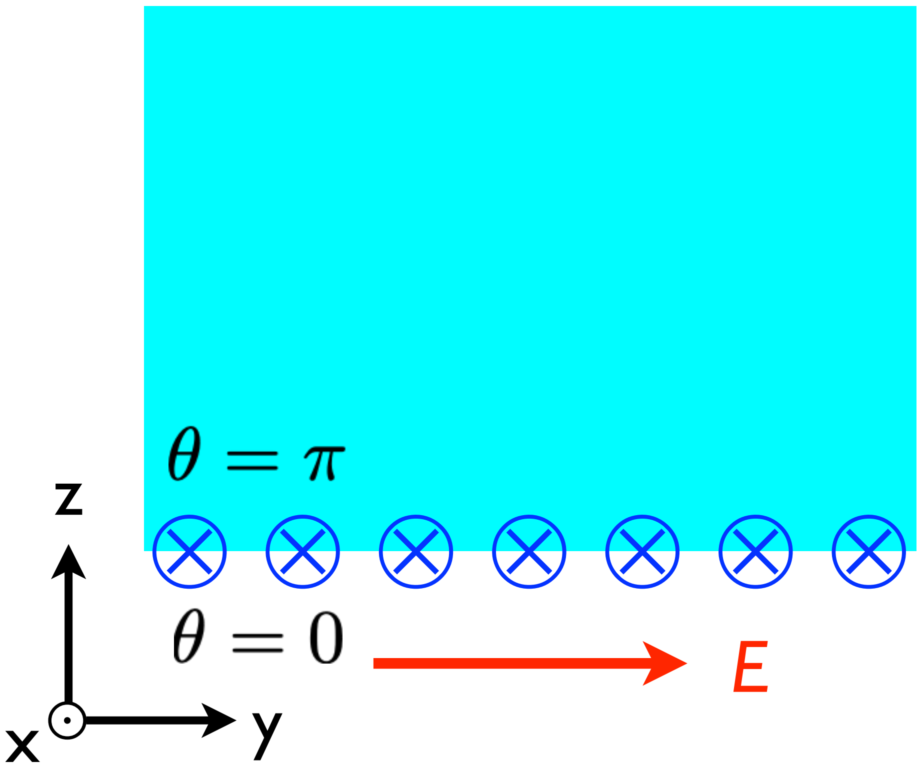
"Half Integer Quantized Hall Effect" on Surfaces



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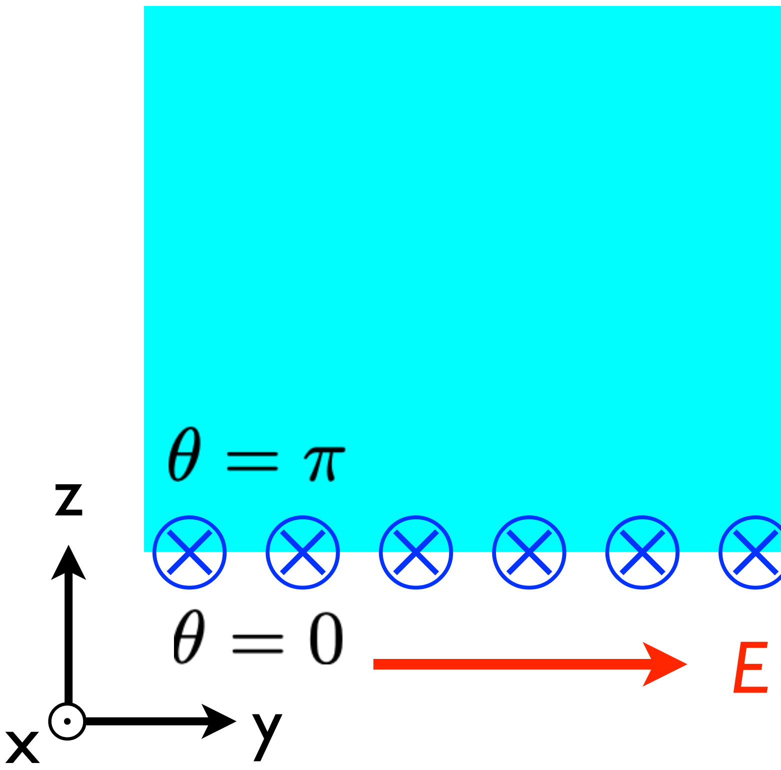

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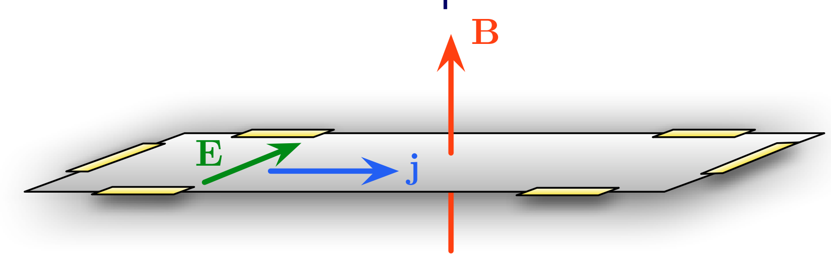

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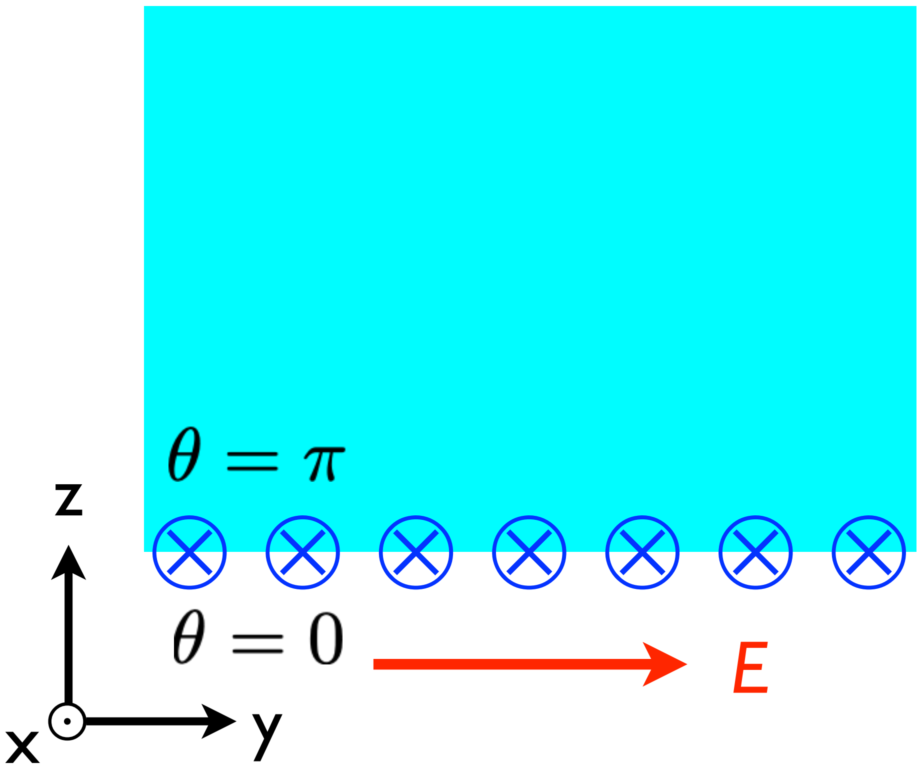

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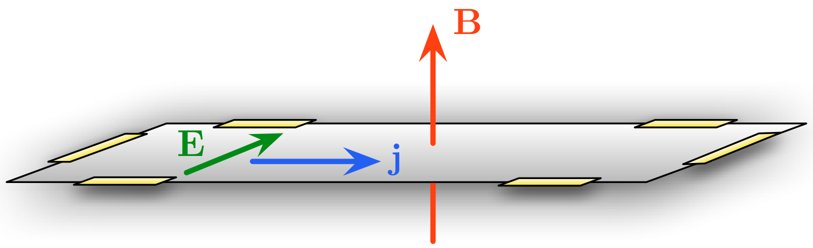

surfaces between insulators with θ=0 and θ=π have a <u>half-integer</u> <u>quantized</u> "surface Hall conductivity"

$$\sigma_{xy} = (-)\frac{1}{2}\frac{e^2}{h}$$

Half-Integer Hall Quantization as a Signature

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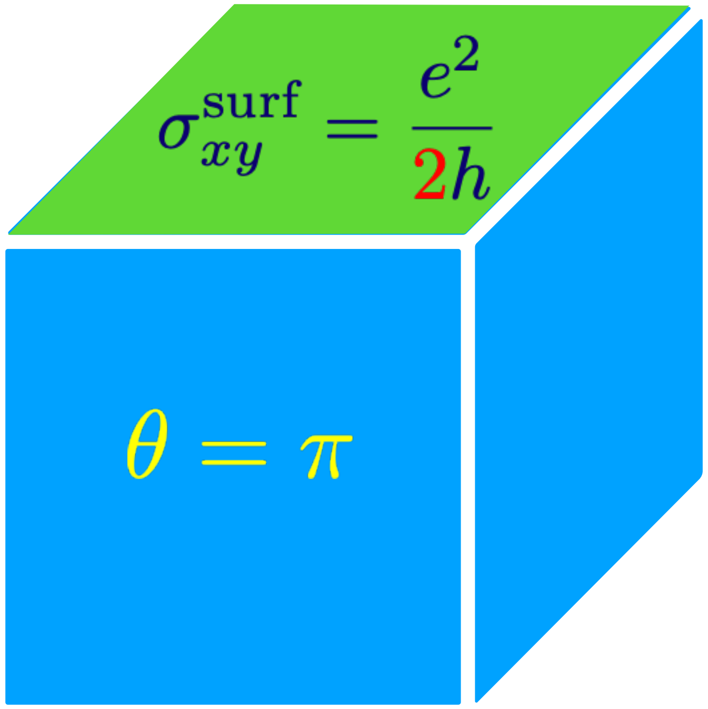

Surface changes near the interface are purely 2D. As such, for weak interactions (assumed here) they can only have*

σ_{xy} = (integer) e^2/h

*The physics of 2D quantized Hall effects is a rich story in its own right, recognized with 3 Nobel Prizes: von Klitzing ('85) ; Tsui, Störmer, Laughlin ('98); Haldane & Thouless ('16)

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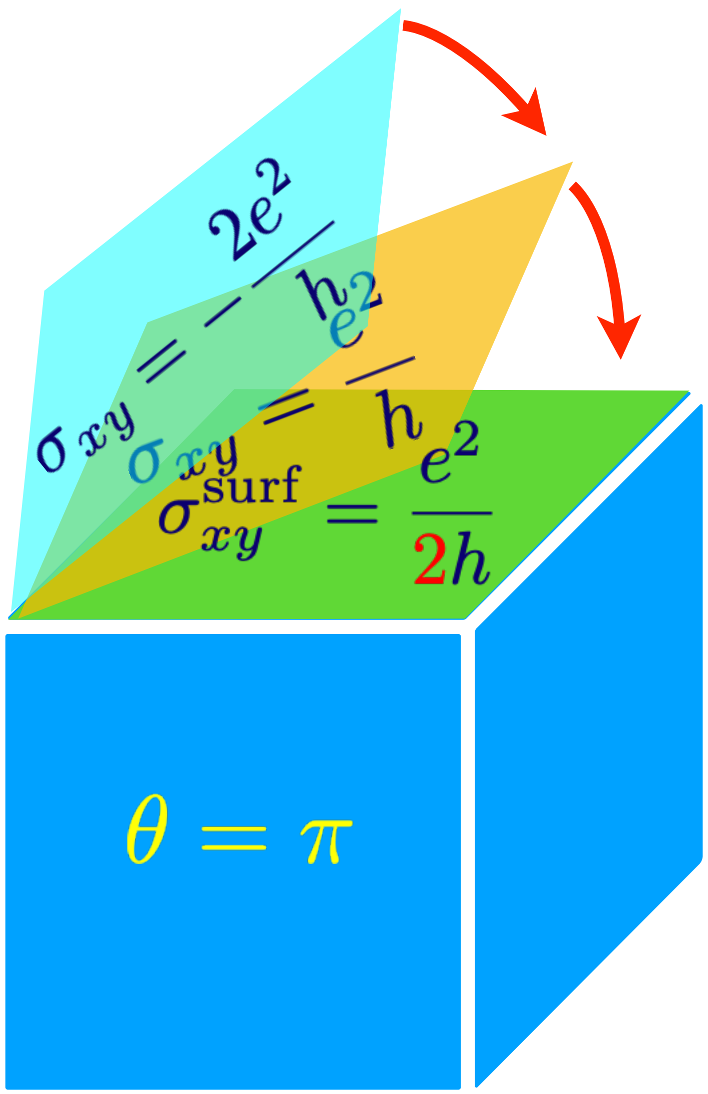

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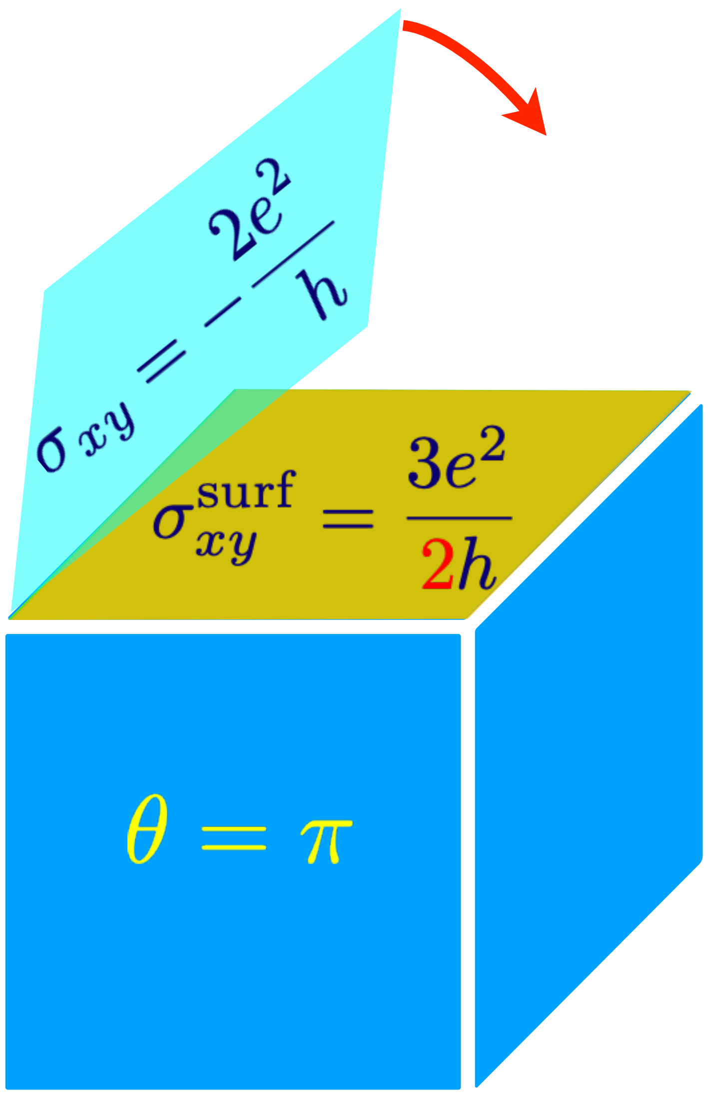

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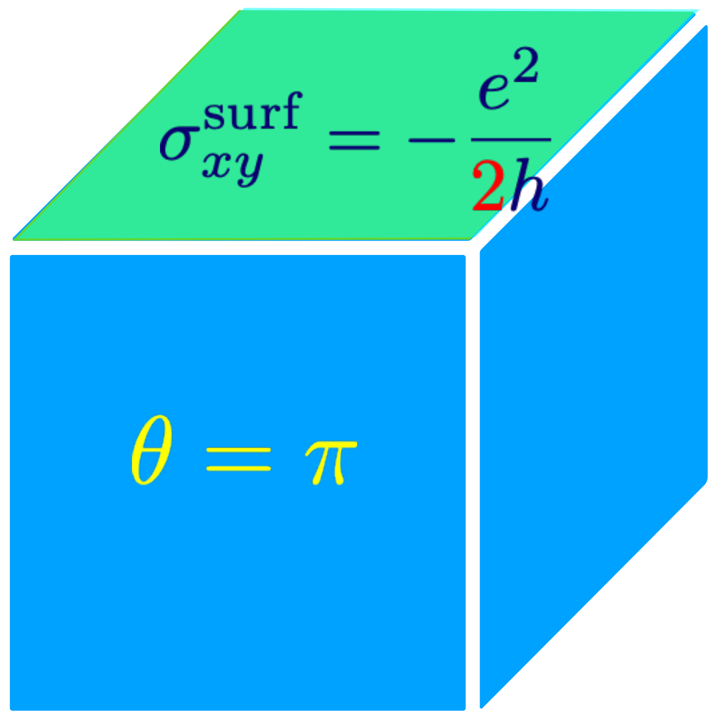

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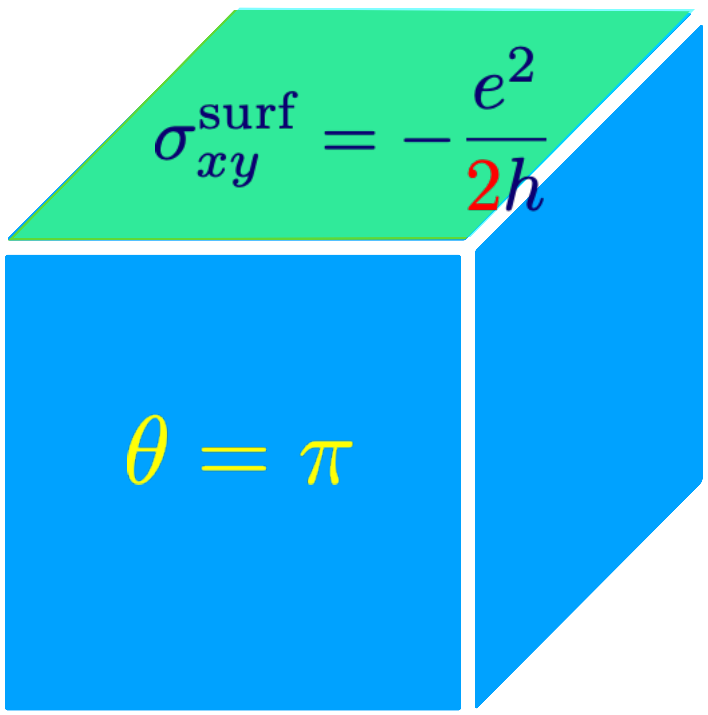

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Viewed as a purely 2D system our θ-interface is "anomalous"
(There is no paradox since it *requires* the third dimension over which θ varies)

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Materials: Topological Insulators

Consider solid with $\theta = \pi$ and fully time-reversal symmetric (incl. surface)

Insulators have $\sigma_{xy} \neq 0$ only if T is broken: contradiction on the surface!

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A T-symmetric system w/ $\theta = \pi$ is a "topological insulator":
A new phase of matter whose interface with a trivial insulator is always metallic
(as long as T is unbroken)

[Prediction in 3D: Moore & Balents '07; Fu-Kane-Mele '07; Qi-Hughes-Zhang '08; Roy* '09]

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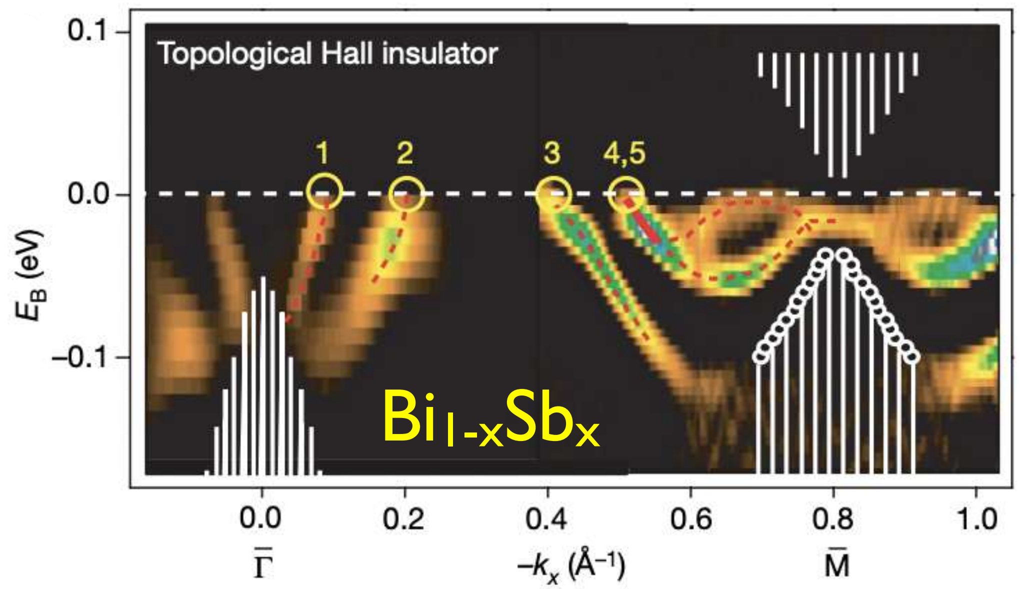
Experimentally detectable b/c surface metals special: host odd # of "Dirac cones"

[Hsieh *et al* Nature 452, 970 '08]



Dispersion of surface bands can be measured by Angle-Resolved Photoemission Spectroscopy (ARPES)

Oxford's Yulin Chen is a world leader in ARPES experiments on topological matter

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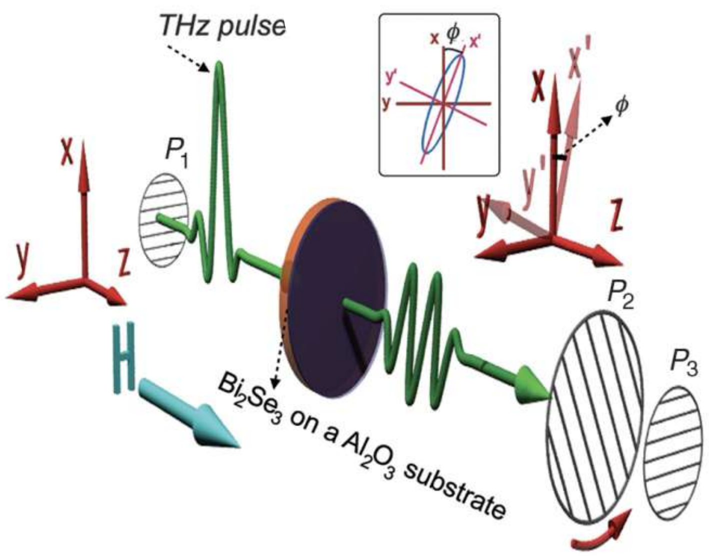
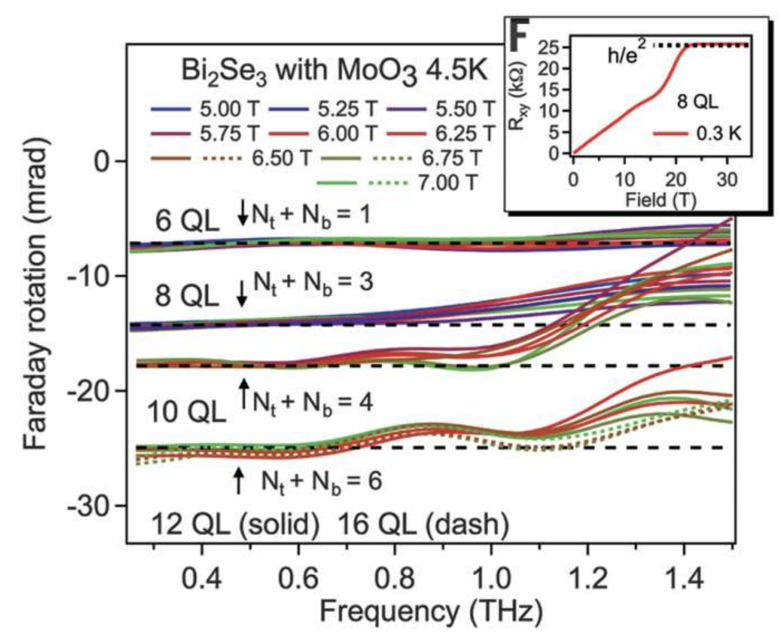
One signature of magnetoelectric effect: quantized "Faraday rotation" of polarized light



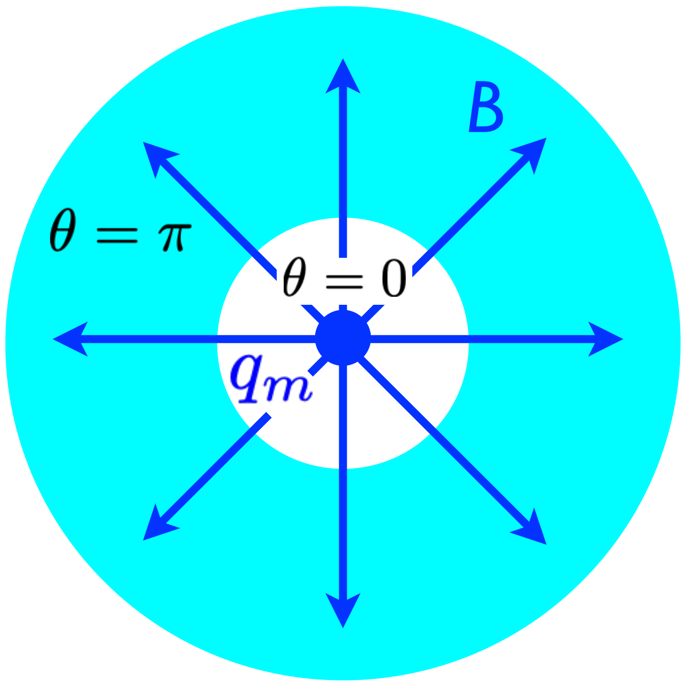
[L.Wu. *et al* Science 354, 1124 '16]



tricky to pin down "halfness" of a *single* surface - much work ongoing to do better

The Witten Effect

Imagine a sphere with θ=0 inside a θ=π region,
and place inside it a point magnetic charge or "magnetic monopole"



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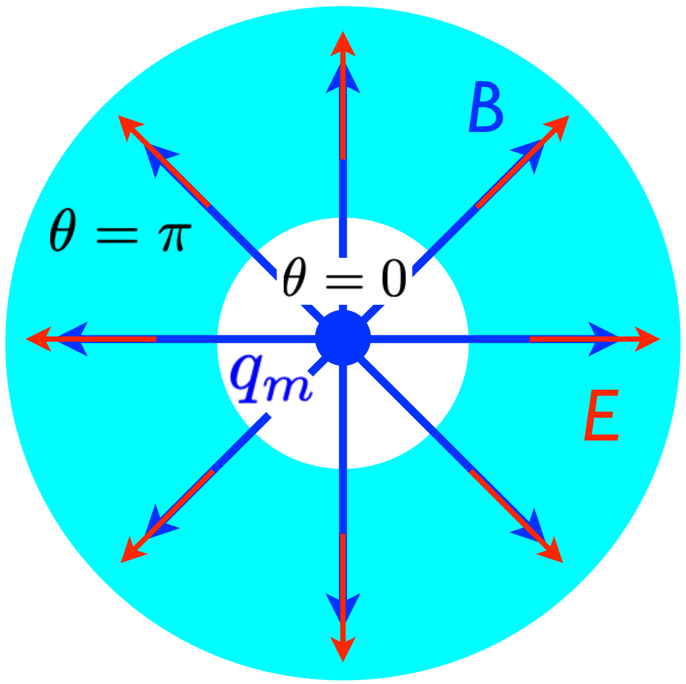

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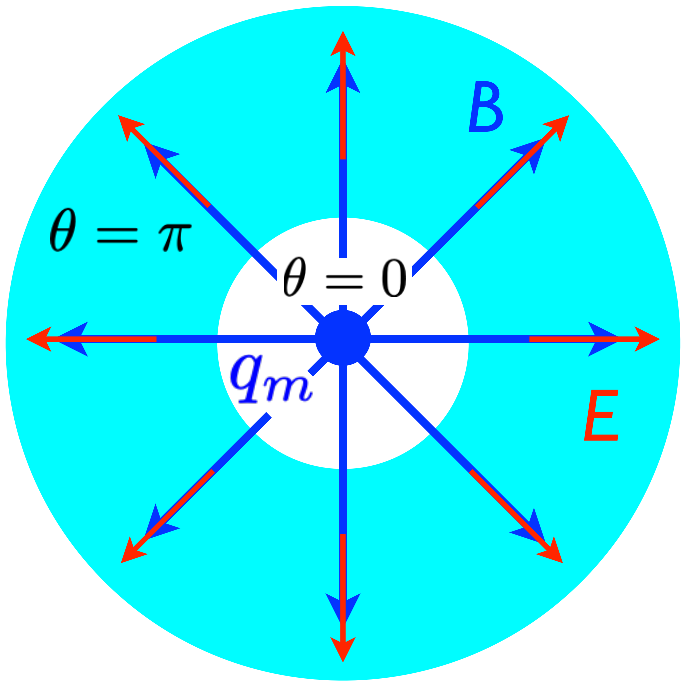

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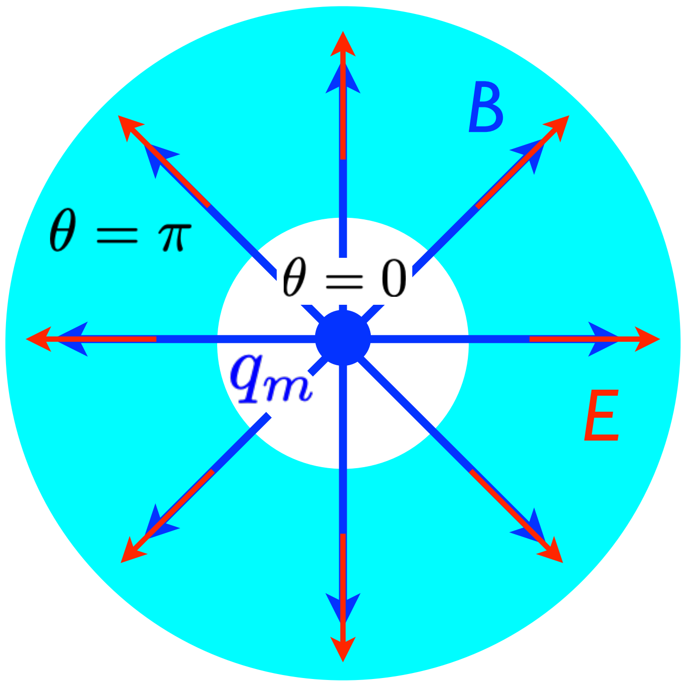
Equivalent to field of a point electric charge $q_e = \alpha q_m$

The Witten Effect

Imagine a sphere with θ=0 inside a θ=π region,
and place inside it a point magnetic charge or "magnetic monopole"



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Argument independent of size* of θ=0 "hole" ⇒ monopoles are "dyons"

*Ultimately linked to boundary conditions on gauge fields at the origin.

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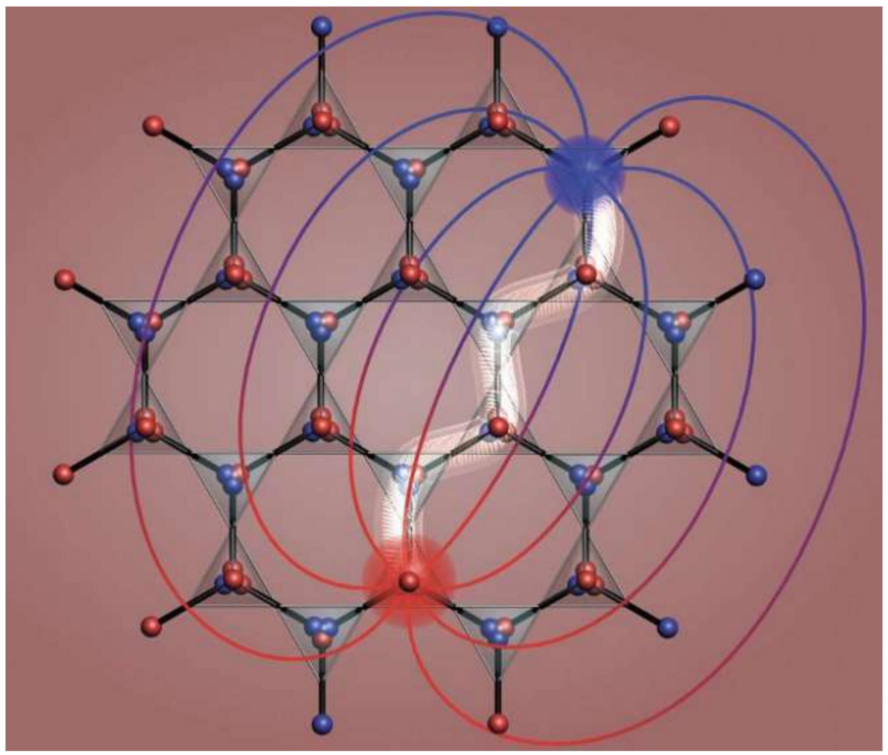

Magnetic monopoles in spin ice

C. Castelnovo[1], R. Moessner[1,2] & S. L. Sondhi[3]   Nature 451, 42 '08

Oxford postdoc, 2006-10

BA '94, DPhil '97, faculty '06-'07

Wykeham Professor, 2021-

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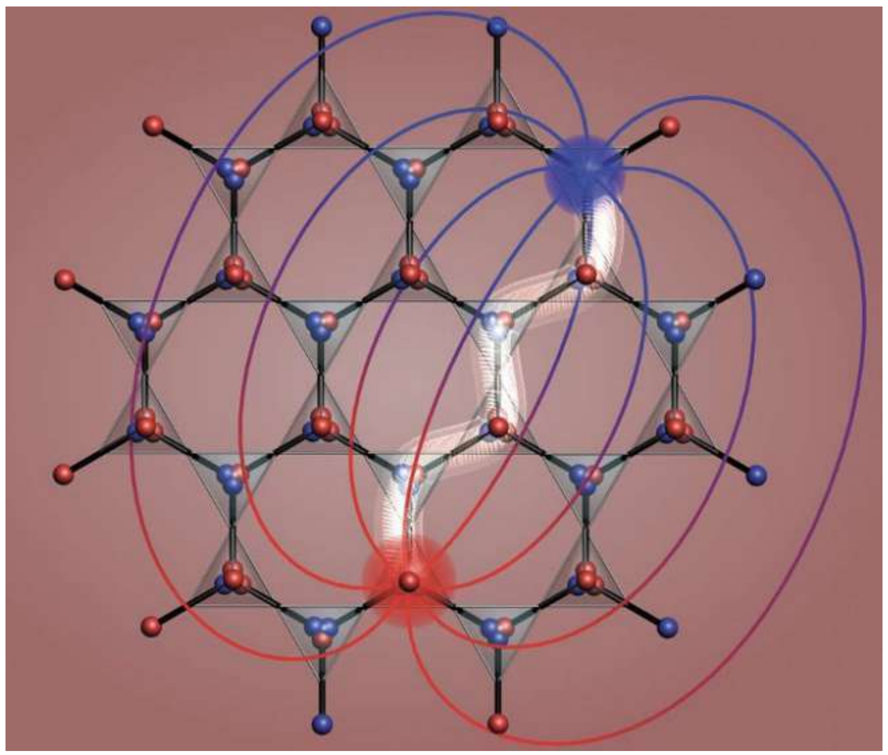

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The "*E*" and "*B*" fields of these monopoles are <u>also</u> emergent (correlated excitations of many spins) and in the right circumstances can also experience θ-terms.

The "Witten effect" of monopoles in this fully-emergent axion electrodynamics could have observable signatures!

Summary

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We now know several insulators whose band topology places them in a new state of matter, an "axion insulator" whose "vacua" includes a θ term

There are many active experimental searches for the exotic phenomena predicted by axion electrodynamics

May be possible to also study the Witten effect, and dynamical axions in solids

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Whether or not axions[1] have any physical reality, their study can be a useful intellectual exercise.

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"It's very difficult to know whether something is useful or not, but one *can* know that it's exciting."

— F.D.M. Haldane, October 4, 2016, on the day of the Nobel announcement